Some reasons why we cannot believe in an analog reality.

It is interesting that even after 2500 years or so we are still having the same debate as to whether we live in an analog or digital world. This cannot be resolved by mathematics. In fact it is our mathematics, miraculous as it is, that is one of the chief contributors to the dilemma. Consider the areas in physics we still cannot really define; motion, inertia, time, energy and gravity. We can describe in exquisite detail what they do but cannot really say what they are.

We often speak of 'pure' energy but actually we never experience such energy. In all cases energy, as we experience it, is structured. The sun, a fission bomb, a fireplace all emit energy as structures, in these cases electromagnetic radiation. All the energy we experience is discrete. Energy, rather than being released is restructured. The wave-particle phenomenon does not change this in any way. Energy then is never 'released' as pure energy. It is simply restructured. The conundrum here then is just what is energy? It would appear that it does not exists as an unstructured entity. It would also seem that energy gives no support to an analog reality.

Then there is motion. In the Doxographists[1] Plato referred to a Pythagorean concept of motion: *motion is a certain otherness in matter.[This is the common limit of all motion.]* If asked, most would say that motion is an analog function, but consider this. In order for an object to be said to have moved it must be in a different place than it was originally. This implies a distance as without a distance the object could not be said to have moved. We can take half that distance and then half again for as many times as we wish but the distance can never become infinite. Otherwise the object would not have moved.

This is another way of saying that an object cannot move by taking infinitely small steps which, if motion is truly analog, it would have to do. Since infinitely small steps are impossible there must be an initial step, however small this might be. The next step must be finite as well so motion is digital because it cannot be otherwise. Some might say that we have studied motion with exquisitely fine instruments and such a step has never been measured. We have never measured a Planck Length either but no one doubts its existence.

This creates another problem. The initial step that an object must take is still a form of motion so we are still faced with the impossibility of taking infinitely small steps. An electron appears to jump from orbit to orbit. Apparently it does not *move* to another orbit. It is here and then it is there. Could macroscopic objects do the same? Since an infinitely small step is a physical impossibility then steps of motion must be something quite different.

I think we forget sometimes that mathematical abstraction goes back at least to Euclid. Look up the definition of a point in any dictionary and you will read that a point is thought of as having a location in space but having no dimensions. Thus a point, by *definition*, does not exist. The same dictionary will quite likely contain the classic definition of a circle; a circle is a curved line, every *point* of which is equidistant to a common center *point*. Thus, *again by definition*, a true circle is a fiction.

If we inscribe a regular polygon inside a circle we can determine its circumference (the polygon) by measuring one side and multiplying it by the number of sides. If we double and redouble the number of sides the polygon begins to approach the circle. But again, as with motion, the number of sides cannot become infinite. We might therefore define a circle as a regular polygon with an infinite number of sides. The same holds true for all curves. They are abstract for the same reason that a circle is.

From this we may infer that all motion is rectilinear. If a moving object were to describe a curve the azimuth of the motion at any given time would always be a line tangent to the curve. Again, as with motion, this azimuth must take a finite step in order to change its direction

In these examples the abstract factor is infinity. The problem with infinity is that it cannot be defined. There simply is no such thing as infinity. We give it a symbol and then deal with it as if it were real. The question - how can an object move through an infinite number of points in a finite amount of time, is a non question. It has absolutely no meaning. It certainly is no paradox as it has sometimes been described.

We often encounter this remark; there are an infinite number of solutions to the equation x + y = 2. There are as many solutions as one might want to develop but the number can never be infinite.

Some years ago Carl Sagan was explaining infinity. He first described a googolplex ($10^{10^{100}}$) He then explained that with either the digit 1 or a googolplex infinity is just as far away from either quantity[2]. The problem with this explanation is that 'far away' implies a distance and a distance implies a finite measure. This is the most common error made when infinity is defined. Since infinity cannot be defined the term 'far away' has no meaning.

If we were to live forever we simply would never die. We would never become infinitely old.

Mathematicians tell us that our math has become sufficiently sophisticated that we now know that the sum of an infinite series ($\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16.}\ldots\ldots\ldots\frac{1}{n}$) equals 1. If that be true, what is the penultimate fraction? If such fraction does not exist then the result of an infinite series simply cannot equal 1. If, as relativity theory suggests, the big bang began with a singularity what is the first definable size; or temperature; or density? To give this initial point mass goes against the theory. Singularities plague our formulations and, as one physicist said, they are dealt with by often sweeping them under the rug.

Irrational numbers were known and used for many, many years before Richard Dedekind proved they are real[3]. What actually did he prove? We still cannot write one down. There exists no number which when multiplied by itself, will equal 2. Regardless of how many places we extend the mantissa of ($\sqrt{2}$) the result of $(\sqrt{2})^2$ will always be less than 2. There is absolutely no reason in the world why there should be a square root of 2.

Einstein[4] often said in various ways that out mathematics does not reflect reality. The things we have created with the various flavors of the calculus is truly remarkable when you consider the calculus is based quite literally upon a grand oxymoron; instantaneous speed. From this of course we got the derivative of a real value function which led to the whole magic of the calculus.

On the other hand when we solve for the area under a curve we are in effect taking the sum of areas of a bunch of very thin rectangles, in other words, in effect, a digital solution. I have always felt, in a manner of speaking, that the calculus consists of doing things that cannot be done to other things that don't exist. This is not to denigrate the calculus. It is often said that the calculus is man's greatest achievement and this could quite likely be true but that does not make it any less abstract.

Some time ago an established scientist was, in a popular science magazine, setting up an example using endless decimals, in this case .999999999999999....The first thing he did was multiply it by 10. He did this by using the base ten shortcut and simply moved the decimal point one digit to the right. So now he had 9.9999999........Grade school arithmetic tells us that in order to multiply a number by 10 you first place the 1 under the last digit of the multiplicand, Now, since there is no final digit in the multiplicand the 1 cannot be placed under the last digit.

Simply put, you cannot use arithmetic operators on an infinite quantity because an infinite quantity cannot be defined. In the real, nuts and bolts world, there are no infinite sets nor are there empty sets. A denumerable infinity is a fiction.

Leon Kronecker vehemently opposed such ideas and for good reason[5]. Kronecker was not in any way besmirching the calculus. He was, in fact, quite skilled in that area. What he was alluding to was that all the conjecturing in the world will not produce a definable infinity.

Then there is the field. Albert Einstein, in another one of his pithy remarks, which are so often mentioned but so often ignored, wrote this in 1954 to his long time friend Michelle Besso[6]:

> *I consider it quite possible that physics cannot be based on the field concept, i, e., on continuous structures. In that case, nothing remains of my entire castle in the air, gravitation theory [and of] the rest of modern physics.*

It is a pity that he did not live to pursue this idea further

If we place a sheet of paper on a magnet and then sprinkle iron filings on the paper the filings arrange themselves in lines. If a field is continuous then why do we see lines? The solar prominences also form what amounts to lines. These are the results of the solar magnetosphere.

The notion of the infinite brings up the continuum. In his Continuum Hypothesis[7] Cantor stated that;

> *The continuum is the conjecture that every infinite subset of the real numbers can be put into one-to-one correspondence with either the set of positive integers or the entire set of real numbers.*

You cannot have in actuality an infinite subset without a definable infinity. This essay is not written to debate set theory or logic or pure math. It is written to offer an proposition that the real world is digital. I think possibly discrete is a better choice of words but that is of no importance.

Our usage of zero is similar to our usage of infinity. Zero is a marker. It is really nothing more. It is very useful in base ten arithmetic. Classing it as an integer is another fiction. This probable came about from the Cartesian coordinates of a standard graph where zero marks the center focal point. The term $x^0 = 1$ is a convention. It has no actually meaning. It is useful but fictitious. It quite likely arose because of the notion that a power series starts with 1. This is becomes obvious when we consider the convention; $x^1 = x$. This sets the value of the power series and the exponent is a way of illustrating this. The power series of 3 begins with 3 and the power series of 5 begins with 5.

Zero is now considered to be an integer, and a positive integer at that. As with infinity zero has no conceptual meaning. An asymptotic series can no more reach zero than it can reach infinity. Kronecker also was disturbed by the notion of negative numbers[8]. Actually negative numbers can be avoided in this fashion;

-5,	-4,	-3,	-2,	-1	0,	1	2	3	4	5
1	2	3	4	5	6	7	8	9	10	11

Both series are equivalent. This is the way that the enharmonic series in music avoids both the zero and negative numbers. Thus the zero as well is not a part of the digital reality.

Defining irrational numbers as real brings up an interesting point. We define an irrational number such as π largely by how we plan to use it. We might state that π = 3.1416. This a very common definition of π. Perhaps we what a bit more accuracy so we define it as π = 3.14159265. These are two different numbers. 3.1416 certainly does not equal 3.14159265.

Supposes we let the decimal run on to a million places. This can and has been done many times. Then we add another decimal place so now we have a million and one decimal places. None of these versions of π equal each other. Also none of them equal π. The number of places we can allow is unbounded. We can keep increasing the mantissa of π to any degree but it can never be infinite.

It seems that everything we encounter in the real world is structured; mass is structured; electromagnetic radiation is structured. Even motion would appear to be structured. The one thing that seems to defy categorizing as structured is time. We cannot stop time as we can motion, however, since time is so closely locked to motion we can entertain the notion that time is structured as well. The instance is a corollary of the point as in motion. It allows us to segment part of motion or a part of time without any intrinsic reality.

Among the paradoxes of Zeno of Elea is the one that states that in order to cross a room you have to first transverse half the distance, then half that distance and so on. This is said to mean that we cannot cross the room. We can state that we have crossed half the distance if, *and only if*, we stop moving. Then we have defined a segment that is truly half the distance, On the other hand if we do not stop there exists no point where we can say we are half way. The distance is always changing. Stopping and restarting the motion requires both an acceleration and a deceleration and these do not exist while and object is moving. There is no paradox.

Everything we encounter in the real world has mass, thus anything that moves has mass. If massless particles do indeed exist that has no bearing on this essay. As particles they are still discrete. This gives another look at motion, structure and mass. If we strike a cue ball exactly in the center and point it toward the exact center of an object ball we can see that the cue ball stops moving instantly and the object ball continues the motion of the cue ball. The cue ball does not decelerate and the object ball does not accelerate. When must have happened is an exchange of structures.

Since nothing in the universe is standing still the object ball has structure which is not unlike the cue ball. Another example is a device called Newton's Pendulum. In this device a series of balls which are consistent as to size and mass are hung by wires from a common horizontal bar so that they touch each other. If a ball on one end is pulled back and released the ball on the opposite end continues the motion of the initial ball just as did the object ball and the cue ball and for the same reason.

It is often said that the energy moves through the balls from the moving ball to the end ball. This says absolutely nothing. What does it mean; *'the energy'*? Changing the size or mass of any of the balls significantly changes the behavior of the device. This indicates structure. If you draw back two balls two balls swing out from the other end. This further illustrates structure.

Is time the result of some other occurrence? Is it self contained? Does it exists at all? It is Omni present? We need it to determine speed, acceleration and the results of our clocks.

Then we come to gravity. Again there is an Einsteinium notion that is often mentioned but apparently universally ignored. Gravity is not a force[9]. It is a condition. You depress the accelerator pedal in your car and you are thrown back into your seat - that's a force.

Einstein used the example of a person jumping from a tall building. There is no sense of anything pushing or pulling that person down. In fact if it weren't for the sight of the building moving past or the rush of air it would be impossible to tell if you were falling or not. We have no way to sense motion. We can only sense acceleration or deceleration. One common belief that appears to be somewhat prevalent in modern thoughts about physics and gravity is the since a gravitation field warps space gravity does not pull you into the chair upon which you are sitting but rather it pushes you.

This is absurd. There is no difference between a push and a pull save for the direction from which it is coming. It is still commonly believed that gravity propagates, and not only that but at the speed of light. This leads to the assumption that if the sun were to suddenly disappear we would not know it for some eight minutes. This I believe to be completely wrong. We would be aware of the disappearance of the sun the very instance it disappeared and I offer a postulate as to why.

It would seem to be more logical to assume that gravity is an inherent part of every atom when it is formed. The atom does not create the gravitational field any more than it creates the electrons that are a part of it. An atom cannot be created without it. The warping of space is a significant part of just what an atom is.

We feel our weight when sitting in a chair, not because we are being pushed or pulled, but by the fact that we are still falling. A body can have any number of motions and all are independent of each other. The moon has more than fifty discrete motions. The earth has a strong gravitational field because it has a great plethora of atoms.

Space itself cannot be compressed so stating that the gravitational field warps the space around it must refer to the gravitation field and this field must be structured. Every object regardless of how large or small has such a gravitational field and the 'warping of space' is quite likely nothing more than a restructuring of the object's gravitational field; any object.

Gravity is omnipresent. It exists everywhere in the universe. Everything that exists in the universe we experience has such a gravitational field. All motion in the universe must be determined by the nature of each object's gravitational field. Those residing the space station, if they were to measure everything they would find no difference from the same things as measured on the earth and yet, to us on the ground, that station is traveling at seventeen thousand miles an hour.

The thing we did to the components of that station was to accelerate them. Since nothing discernable changed we can only assume that what we did change was the station's gravitational field. It is quite likely the interaction of gravitational fields that governs the nature of motion. Since every object in the universe does not sense its motion perhaps all objects are moving at the same speed, the apparent different speeds when viewed from another body with a different gravitational structure is caused by the different gravitational structure of the body being viewed and the gravitational field trough which it is moving.

Since gravity diminishes according to the inverse square law we could assume that the elements of the gravitational structure become closer together as the field approaches the body in question. If this be true then an object moving at its basic speed would appear to accelerate because it would take less time to navigate each structural segment.

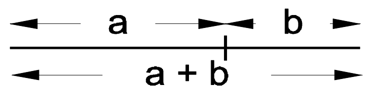
Time is present in all of these discussions. By stating that the space station is traveling at seventeen thousand miles an hour includes time as a determining factor. Since time is a part, and quite likely an integral part, of so many of real world phenomena it can be argued that time itself, if it does exists, is also discrete.

For a final example we shall take a look at another irrationality, the Golden Ratio. The Golden Ratio is another linearization of a discrete series. First of all it is not a ratio. While we do at times use a rational approximation for an irrational number, such as using $\mathbf{22/7}$ for π**,** we never make the mistake of calling π a ratio. The Golden Ratio is typically symbolized by the Greek letter φ which, as with π**,** gives it a reality that it really does not possess.

In mathematical terms φ is expressed thus:

$$\varphi = \frac{\mathbf{1 + \sqrt{5}}}{\mathbf{2}} = 1.6180339887498948482045868343656 4$$

Geometrically it is expressed thus:



a + *b* is to *a* as *a* is to *b*

Simple algebra shows us that φ is irrational.

$$\frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{\boldsymbol{b}}{\boldsymbol{a-b}}$$

If a/b is a fraction in lowest terms, then $b/(a-b)$ is in even lower terms — a contradiction.

While it is a fiction it is indeed a fascinating fiction. Consider this series;

$\varphi = \varphi$

$\varphi^2 = \varphi + 1$

$\varphi^3 = 2\ \varphi + 1$

$\varphi^4 = 3\ \varphi + 2$

$\varphi^5 = 5\ \varphi + 3$

$\varphi^6 = 8\ \varphi + 5$

The integers in both column to the right of the equal sign show the Fibonacci Series. This series is formed by using as the next integer the sum of the two previous integers, thus

3, 5, 8, 13, 21, 34 etc.

Every pair, taken as ratios, forms an approximation of the Golden Ratio. Starting with the innermost rectangle the decimal values of the successive ratios $\mathbf{1/1}, \mathbf{2/1}, \mathbf{3/2}, \mathbf{5/3}, \mathbf{8/5} \ldots\ldots\ldots\ldots \boldsymbol{w/h}$ (w = width and h = height). This presents an interesting pattern.

$$\frac{5/3}{8/5}$$
1.6666…
1.6

$$\frac{8/5}{13/8}$$
1.6
1.625

$$\frac{13/8}{21/13}$$
1.625
1.615384615384615384615384615384615384 6

$$\frac{21/13}{34/21}$$
1.6179775280898876404494382022471 9
1.619047619047619047619047619047 6

It must be remembered that defining the ratios as decimals is the same as defining them as intervals, as would done with sound. 1.625 does not really define the rectangle, 13:8 does. The same is true for intervals and frequencies. The basic shape of these pairs is that the top number is in one instance the numerator is greater than the denominator and in the next the denominator is greater than the numerator. The pattern 1.6 and 1.66666.., in a manner of speaking, marks the front end of the series, as every successive pair is a small and progressive deviation from the ratios of this second pair.

The ratios of each successive pair becomes a mathematical series that converges to φ. This series is asymptotic. φ is a true abstraction.

For those who prefer formulas the series looks like this;

$$\sum_{n=1}^{\infty} | F(n)\varphi - F(n+1) | = \varphi$$

If we take the ratio of any given pair, such as

$$5/3 = \mathbf{1.66666666}$$

The difference between that ratio and the Golden Ratio is 1.030056647916491413674311390.

$$\mathbf{1.66666666} / \mathbf{1.61803398874} = \mathbf{1}.0300566479164914136743113906094$$

This is the greatest error between the Golden Ratio and the ratios of the Fibonacci Series.

Even in small structures it would be virtually impossible to distinguish between the Fibonacci ration and the Golden Ratio. Each successive ratio becomes closer to φ. As with π we can come as close to φ as we might wish. No architect in his right mind would attempt to use the Golden Ratio as a definition of a rectangle, the so called Golden Rectangle. It has been said that the Parthenon is an example of use of the Golden Ratio.

It is doubtful if either the chief architect, Ictinus not his assistant Callicrates used the Golden Rectangle as the measurement system for the various rectangles we see in the Parthenon. These rectangles exhibit the Fibonacci series as well and quite likely that is what they used, or perhaps they used a single pair for all of the rectangles.

. Vitruvius, whose knowledge was drawn from both Greek and Roman architecture, never mentions it in the ten books$_{10}$. If it had been a tool of the ancient architects one would think he would at least referred to it. The ten books of Vitruvius were the Bible for all architects during the Middle Ages up until the Renaissance.

The Golden Ratio then becomes an irrational replacement for the natural series that comes from the Fibonacci series. The same thing occurs in music. In any key in music there are thirteen tonalities, not twelve. Thirteen is needed to complete the structure of the circle of fifths. If we are in the key of C we need both F♯ (1.40625) and G♭ (1.4222222....) to properly define what is known as the circle of fifths. This is the prime enharmonic change. On a keyboard both pitches are represented by one key and, in even-tempered tuning this is represented by $\sqrt[2]{2}$ (1.4142135623730). This value sets right between F♯ and G♭ just as does the Golden Ratio which sets right between each Fibonacci pair.

We often hear about the Golden Spiral which is said to be found often in natural structures. Nature does not construct things with irrational numbers simply because it can't. The Fibonacci spiral is indistinguishable from the abstract Golden spiral and we can only assume that is the basis of the natural spiral.

The premise upon which this essay is based is that there is no such thing as infinity, not even as a concept.

Some years ago I had a unique relationship with a remarkable five-year old girl. She related to me as an equal, not as an adult nor as an authority figure. She had an marvelous ability to analyze things. She discovered the word infinity and was intrigued by it.. When I told her there was no such thing and you simply could not count to infinity she initially rejected the idea.

Sometime later she again decided to discuss infinity. She told me that you can count to infinity. "You would have to start counting now and count for the rest of your life. Then you would have to keep counting even after you died. You would have to count for" --her mouth dropped open and her eyes widened as the realization sunk in.. "you would have to count for infinity years." I reminded her that there was no such thing as infinity. "O", she said "I wish there was".

A lot of people have the same wish.

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