When we think of the Fibonacci Series we think of the Golden Ratio.

By the golden ratio I mean

$$\frac{1+\sqrt{5}}{2} = 1.6180339887498948482045868343656 4...$$

Many erudite proofs exist to establish the fact that this number is indeed irrational. Thus by definition it cannot be a ratio, even a golden one. It is an interesting abstraction but I think it has been misinterpreted. We will observe convention and use the designation,

$$\frac{1+\sqrt{5}}{2} = 1.61803398874989484820458683436564... = \varphi$$

Much has been made of the fact that

$$\frac{1}{j} = j - 1 \text{ and } \frac{1}{j-1} = j$$

This looks much more impressive of we use the calculator values

$$\frac{1}{1.61803398874989484820458683436564} = 0.6180339887498948482045868343 65638$$

This, even though it is fictional, leads to fascinating mind games. The danger comes when we accept φ as having a discrete existence. Also, the ratios of successive pairs of Fibonacci numbers seem to converge to φ. Granted, this makes the Fibonacci series most a interesting abstract tool but I think all interpretation, for this discussion, must stop there. When we try to relate this convergence to the real world we run into difficulties.

At first glance the concept of the Golden Section would seem to have litle to do with music. The Golden Ratio likewise has no specific musical meaning. It is just as alien to music as is the tempered scale and for precisely the same reasons. However, we are defining geometric structures by the ratios of their sides and ratios are the building blocks of music.

The notion of a Golden Section is largely a visual thing. Present thinking concludes that the ideal proportions of any rectangle require the ratio of the width to the height must be φ. We are told that many constructions we idealize use the Golden Ratio and the ratio appears very often and in a variety of areas. The Parthenon springs to mind first. It is magnificent, we are told, because the width divided by the height equals 1.61803398874989484820458683436564. Is the perception of the mind really that good? Does the mind in any way interpret the proportions of a rectangle by deriving the ratio of the sides?

In the Ten Books on Architecture written by Vitruvius in the first century the Golden Ratio is scarcely mentioned. It has been a few years since I have read them but I do not remember any mention of the Golden Ratio or the Golden Segment. The numbers Vitruvius does mention are those simple ‘musical’ numbers like 5 and 3. These are the numbers they found in Notre Dame. If we were to construct a rectangle with sides equal to 5 and 3 the ratio produced would be 1.6666…. This is *very* close to the Golden Ratio. This fact has not gone unnoticed and, in fact, is mentioned often in treatises about the Golden Ratio. Unfortunately we are glued to φ just like we are glued to the notion the concert A must be 440Hz. This is *the pitch*.

The difference between $5/3$ and φ is

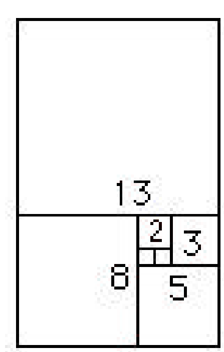
1.0300566479164914136743113906094.

This is a very small difference. As a musical interval it lies between the enharmonic comma (1.024) and the small minor third (1.04166666666667). Audible? – to some perhaps. Visual? – I doubt it.

Let us assume the dimensions of the front end of the Parthenon are equal to 50 feet and 30 feet. It would look like the Parthenon. The non-Golden Ratio would be 1.6666… Supose we want this front end to produce the Golden Ratio. One method of achieving this would be to make the height a hair over ten inches higher. The result would not look conspicuously different from the 50’ x 30’ version. If we were to adjust both the height and width simultaneously the change would be only a couple of inches.

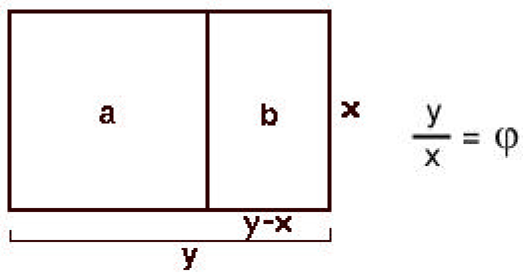
5 and 3 as ratios would have been discovered when the Ancients worked with acoustics (mentioned extensively by many writers). Logic insists that they used 5 and 3 just as does anyone else. The fact that the ratios between successive pairs of the Fibonacci series converge to φ would have been meaningless to them. They were totally unaware of the Fibonacci Series and the notion of convergence. They would have used 3 and 5. If indeed the Greeks used the geometry of the golden ratio it was quite probably another method of achieving $5/3$.

Consider it geometrically. 3 and 5 are part of the geometric construction that defines the Fibonacci numbers. Here is the series graphically.



The smallest rectangle is the innermost one, the one made up of two squares. $1/1$ is the beginning of all ratios. The next rectangle is h = 1 and w = 2. It is also the first two numbers of the Fibonacci series (we will not use zero as the beginning of the series as it makes no sense graphically.)

The Golden Ratio, on the other hand, is defined is defined by $\frac{1+\sqrt{5}}{2}$ or graphically:



A thought about ratios. The fact that $5/3 = 1.6666....$, is, structurally or visually speaking, rather meaningless. What these decimals can provide is shorthand for manipulating these fractions. Apart from this there are no intrinsic properties in these decimals. Starting with the innermost rectangle the decimal values of the successive ratios ($1/1, 2/1, 3/2, 5/3, 8/5$ .... w/h) present an interesting pattern.

(1.0)
2.0
1.5

1.6666…
1.6

1.625
1.615384615384615384615384615384 62

1.619047619047619047619047619047 62
1.617647058823529411764705882352 94

1.618181818181818181818181818181 82
1.617977528089887640449438202247 19

This is an asymptotic series that really begins with 2.0, as 1.0 merely defines the unit length. It must be remembered that defining the ratios as decimals is the same as defining them as intervals, as would done with sound. 1.625 does not define the rectangle, $13 x 8$ does. The same is true for intervals and frequencies. The first pattern, 2.0 and 1.5, defines the basic shape of the Fibonacci pairs, one high then one low. The pattern 1.6666.. and 1.6, in a manner of speaking, marks the end of the series, as every successive pair is a small and progressive deviation from the ratios of this second pair. The top of every pair becomes progressively smaller than $1.6666\ldots$ and every bottom member becomes progressively larger than 1.6. Now mathematics takes over completely and these decimals take on a meaning that they really do not have. The members of the successive pairs are coming, numerically, closer together. These numbers now become a mathematical series that converges to φ. The changes on both the top of pair and the bottom are both asymptotic. φ is a true abstraction.

For instance, a function that expands to ∞ in no way makes ∞ real. The same is true with φ. In truth, $1/j$ is just as physically impossible as $1/\infty$ and for the same reasons.

The problem with choosing φ as the Golden Ratio arises because we see the decimals as objects instead of markers for the height and width of successive rectangles. This is the way we do things mathematically and in this case we are looking through the wrong end of the telescope. We are looking at this convergence absolutely backwards. We can use these decimals to define the amount of change between each progressive pair, for instance, the ‘distance’ between 1.625 and $1.6153846153846153846153846153846\overline{2}$, which is

$$\frac{1.625}{1.61538461538461538461538461538462} = 1.00595238095238095238095238095276$$

If we chart these we get

$\frac{5}{3}$

1.04166666666666666666666666666667

$\frac{8}{5}$

0.984615384615384615384615384615385 (1.015625)

$\frac{13}{8}$

1.00595238095238095238095238095238

$\frac{21}{13}$

0.997737556561085972850678733031674 (1.00226757369614512471655328798)

$\frac{34}{21}$

1.0008658008658008658008658008658

$\frac{55}{34}$

0.99966953073364177131526768010575 (1.0006612663069462468410627689)

$\frac{89}{55}$

1.00012626262626262626262626262626

$\frac{144}{89}$

This series converges to 1. This makes sense, as the Fibonacci Series is a set of ratios whose numerator and denominators are increasing in value. As the fractions progress they approach x/x. As they increase the ratio between them converges to 1.0. We can now interpret the Golden Section and the Golden Ratio from a different viewpoint. The convergence of successive ratios to φ describes successive moves away from something not toward it. The truly remarkable thing about the series is that the every successive pair is a slight deviation from the pair, $5/3$ and $8/5$. This deviation is always so small that the eye would see every Fibonacci rectangle as looking like $5/3$ and $8/5$. This is the magic pair.

We have a Greater Golden Ration ($5/3$) and Lesser Golden Ratio ($8/5$). The Golden Ratio is either $1.666666\ldots$ 1.6, not an abstract point between them. $5/3$ would be the preferred version simply because it is made up of smaller numbers. **Ockham's Razor!**

The mere fact that we are dealing with the numbers 3 and 5 causes us to take a look at music. Structurally the Fibonacci Series stops at 8. Consider our 50' x 30' Parthenon. It is now $5/3$. If we altered the dimensions to 49' x 31' we would get a ratio very close to $8/5$. It would take a keen eye to distinguish between $5/3$ (50' x 30') and $8/5$ (49' x 31'). Of all the ratios in the series this has the most detectable difference. Each successive change is less than it predecessor. Starting with $5/3$ every Fibonacci rectangle looks to the eye as either $5/3$ or $8/5$. Some similarity between graphics and music can be seen when we express both as decimals. 13 does not enter the graphic system as it produces 1.625 and both $5/3$ and $8/5$ produce more useful ratios. The same is true in music for the same mathematical reasons. In a scale with fundamental = 1, 13 produces the note 1.625. The system already has the more useful $5/3 = 1.6666..$ or $8/5 = 1.6$.

In music the ear simply will not tolerate anything above 8 in Fibonacci Series. The entrance of 13 into the enharmonic system creates dreadful beats, which, in turn, spawn even more dreadful second-generation differential tones. The higher frequencies become even worse. On the other hand it would seem that the eye is not so meticulous. I suspect that the eye accepts everything from $5/3$ to that nebulous fraction that produces φ and probably interprets it the same, quite likely as $5/3$. Indeed, would $5/3$ really be more pleasing to the eye than $89/55$? Is a ratio even part of the brain's interpretation? I am guessing that it is and the brain prefers $5/3$.

Another interesting convergence arises if we explore the ratios formed between $^{5}/_{3}$ and each succeeding Fibonacci pair.

$$\frac{\frac{5}{3}}{\frac{13}{8}} \quad 1.02564102564102564102564102564103$$

$$\frac{\frac{5}{3}}{\frac{21}{13}} \quad 1.03174603174603174603174603174603$$

$$\frac{\frac{5}{3}}{\frac{34}{21}} \quad 1.02941176470588235294117647058824$$

$$\frac{\frac{5}{3}}{\frac{55}{34}} \quad 1.03030303030303030303030303030303$$

$$\frac{\frac{5}{3}}{\frac{89}{55}} \quad 1.02996254681647940074906367041199$$

This converges to 1.0300566479164914136743113906094 which is the difference between $^{5}/_{3}$ and φ. This gives a better illustration of the nature of this convergence.

I want to do some work on the Fibonacci Spiral, which to nobody's surprise is not a spiral (at least as commonly defined). In the first place it is a discrete set of rational values. Letting these points define a curved line is a superfluous step. We only need the actual points that are group as a spiral. Connecting the points show us all the possible points and we do not need *all* the points.